

Modelo del Motor D.C

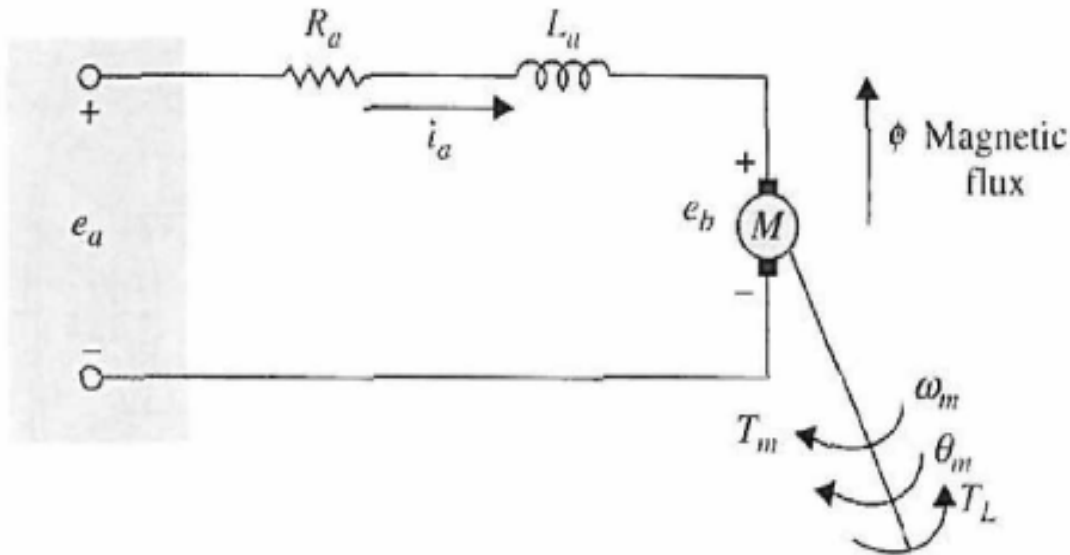


Figure 4-70 Model of a separately excited dc motor.

$i_a(t)$ = armature current

R_a = armature resistance

$e_b(t)$ = back emf

$T_L(t)$ = load torque

$T_m(t)$ = motor torque

$\theta_m(t)$ = rotor displacement

K_i = torque constant

L_a = armature inductance

$e_a(t)$ = applied voltage

K_b = back-emf constant

ϕ = magnetic flux in the air gap

$\omega_m(t)$ = rotor angular velocity

J_m = rotor inertia

B_m = viscous-friction coefficient

Modelo del Motor D.C

$$\frac{di_a(t)}{dt} = \frac{1}{L_a} e_a(t) - \frac{R_a}{L_a} i_a(t) - \frac{1}{L_a} e_b(t) \quad (4-204)$$

$$T_m(t) = K_i i_a(t) \quad (4-205)$$

$$e_b(t) = K_b \frac{d\theta_m(t)}{dt} = K_b \omega_m(t) \quad (4-206)$$

$$\frac{d^2\theta_m(t)}{dt^2} = \frac{1}{J_m} T_m(t) - \frac{1}{J_m} T_L(t) - \frac{B_m}{J_m} \frac{d\theta_m(t)}{dt} \quad (4-207)$$

where K_i is the **torque constant** in N-m/A, lb-ft/A, or oz-in/A.

Potenciómetro

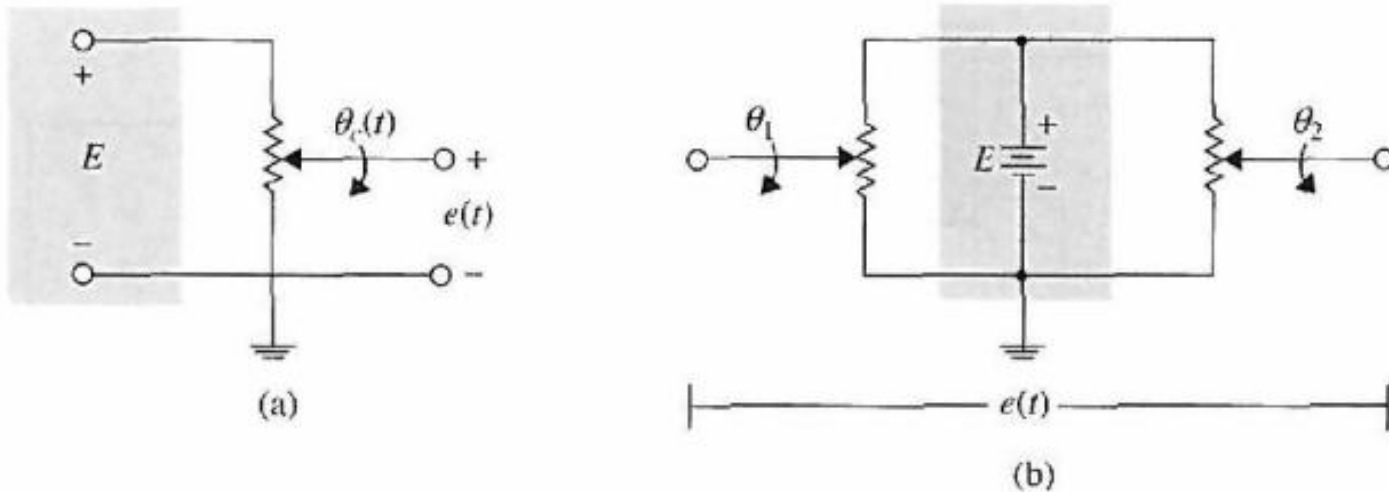
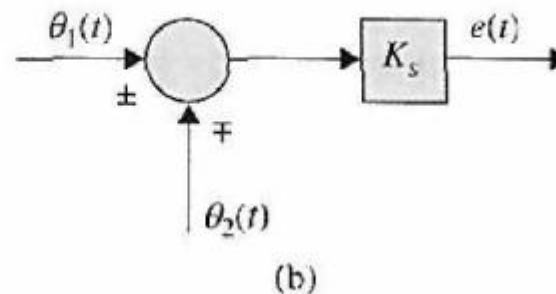
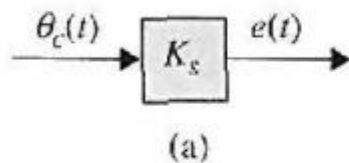


Figure 4-52 Potentiometer used as a position indicator. (b) Two potentiometers used to sense the positions of two shafts.

$$e(t) = K_s \theta_c(t)$$

$$e(t) = K_s [\theta_1(t) - \theta_2(t)]$$



Tacómetro

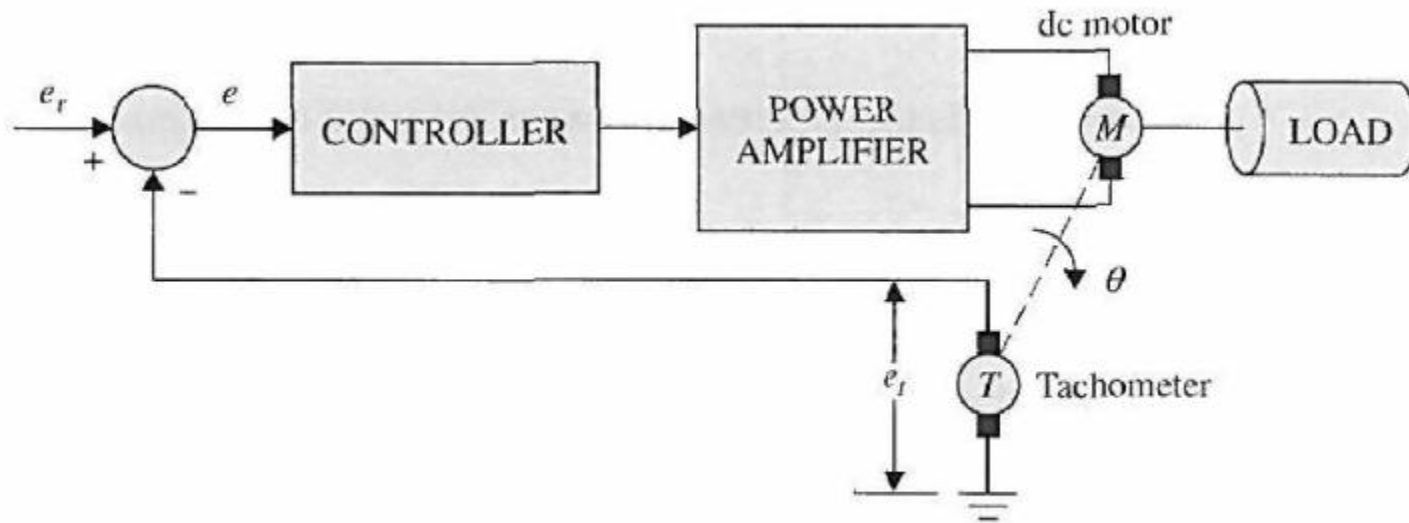
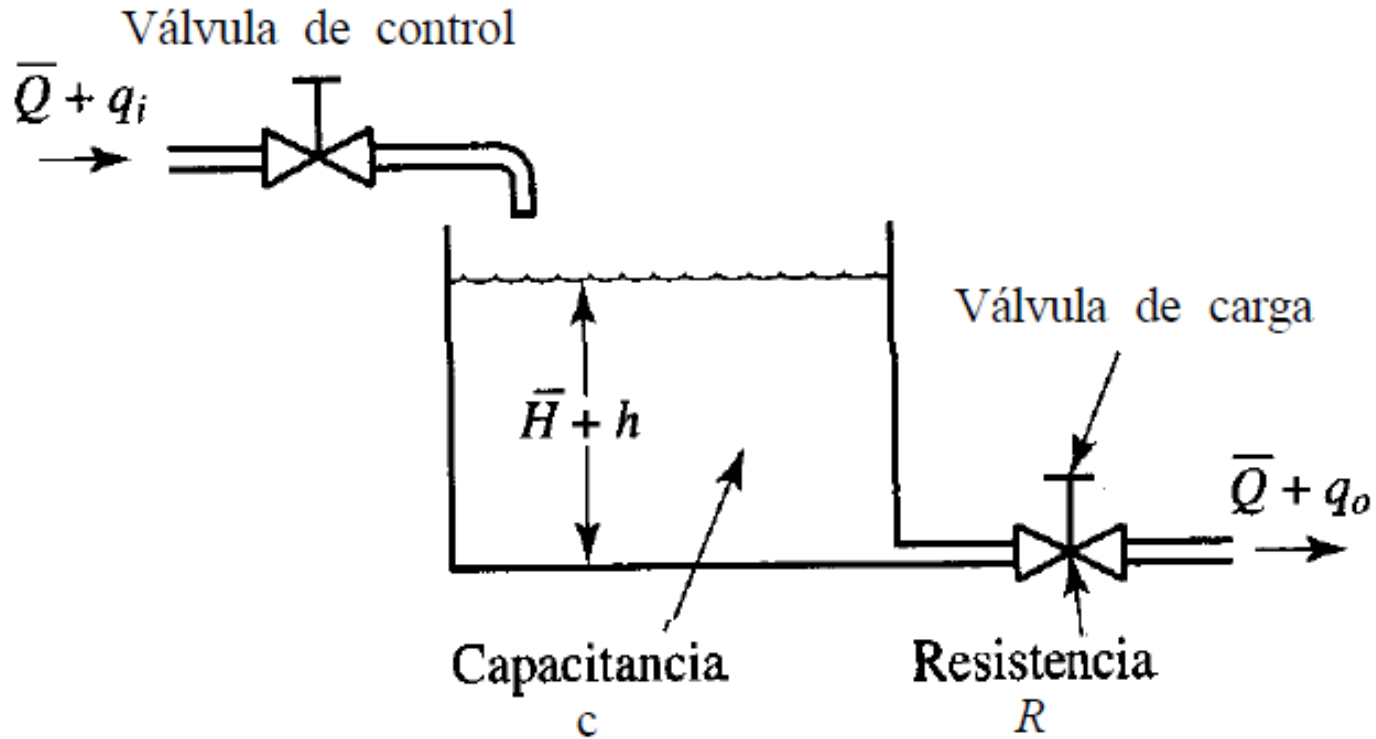


Figure 4-56 Velocity-control system with tachometer feedback.

$$e_f(t) = K_f \frac{d\theta(t)}{dt} = K_f \omega(t)$$

Sistemas de Nivel de Líquido



Sistemas de Nivel de Líquido

Sistemas del nivel de líquido. Considere el sistema que aparece en la figura 3-22(a). Las variables se definen del modo siguiente:

\bar{Q} = velocidad de flujo en estado estable (antes de que haya ocurrido cualquier cambio), m^3/seg

q_i = desviación pequeña de la velocidad de entrada de su valor en estado estable, m^3/seg

q_o = desviación pequeña de la velocidad de salida de su valor en estado estable, m^3/seg

\bar{H} = altura en estado estable (antes de que haya ocurrido un cambio), m

h = desviación pequeña de la altura a partir de su valor en estado estable, m

$$R = \frac{\text{cambio en la diferencia de nivel, m}}{\text{cambio en la velocidad de flujo, } \text{m}^3/\text{seg}}$$

$$C = \frac{\text{cambio en el líquido almacenado, } \text{m}^3}{\text{cambio en la altura, m}}$$

Sistemas de Nivel de Líquido

$$Cdh = (q_i - q_o) dt$$

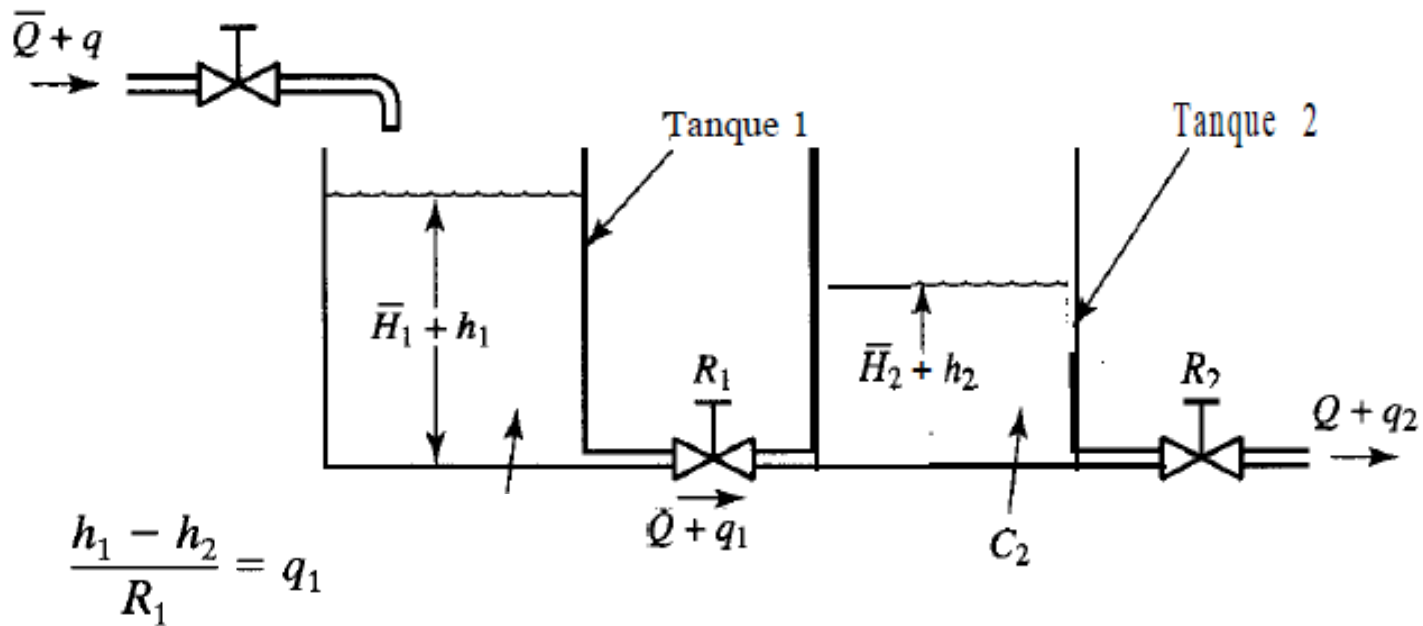
A partir de la definición de resistencia, la relación entre q_o y h se obtiene mediante

$$q_o = \frac{h}{R}$$

La ecuación diferencial para este sistema para un valor constante de R se convierte en

$$RC \frac{dh}{dt} + h = Rq_i \quad (3-69)$$

Sistemas de Nivel de Líquido



$$\frac{h_1 - h_2}{R_1} = q_1$$

$$C_1 \frac{dh_1}{dt} = q - q_1$$

$$\frac{h_2}{R_2} = q_2$$

$$C_2 \frac{dh_2}{dt} = q_1 - q_2$$

Linealización

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{f}[\mathbf{x}(t), \mathbf{r}(t)]$$

$$\Delta \dot{x}_i = \sum_{j=1}^n \left. \frac{\partial f_i(\mathbf{x}, \mathbf{r})}{\partial x_j} \right|_{x_0, r_0} \Delta x_j + \sum_{j=1}^p \left. \frac{\partial f_i(\mathbf{x}, \mathbf{r})}{\partial r_j} \right|_{x_0, r_0} \Delta r_j \quad (4-237)$$

Eq. (4-237) may be written in vector-matrix form:

$$\Delta \dot{\mathbf{x}} = \mathbf{A}^* \Delta \mathbf{x} + \mathbf{B}^* \Delta \mathbf{r} \quad (4-238)$$

Linealización

$$\mathbf{A}^* = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \dots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$$

$$\mathbf{B}^* = \begin{bmatrix} \frac{\partial f_1}{\partial r_1} & \frac{\partial f_1}{\partial r_2} & \dots & \frac{\partial f_1}{\partial r_p} \\ \frac{\partial f_2}{\partial r_1} & \frac{\partial f_2}{\partial r_2} & \dots & \frac{\partial f_2}{\partial r_p} \\ \vdots & \vdots & \dots & \vdots \\ \frac{\partial f_n}{\partial r_1} & \frac{\partial f_n}{\partial r_2} & \dots & \frac{\partial f_n}{\partial r_p} \end{bmatrix}$$

Linealización

$$\Delta x_i = x_i - x_{0i} \quad (4-233)$$

and

$$\Delta r_j = r_j - r_{0j} \quad (4-234)$$

Then

$$\Delta \dot{x}_i = \dot{x}_i - \dot{x}_{0i} \quad (4-235)$$

Since

$$\dot{x}_{0i} = f_i(\mathbf{x}_0, \mathbf{r}_0) \quad (4-236)$$

Linealización

In Example 4-9-1, the linearized system turns out to be time-invariant. As mentioned earlier, linearization of a nonlinear system often results in a linear time-varying system. Consider the following nonlinear system:

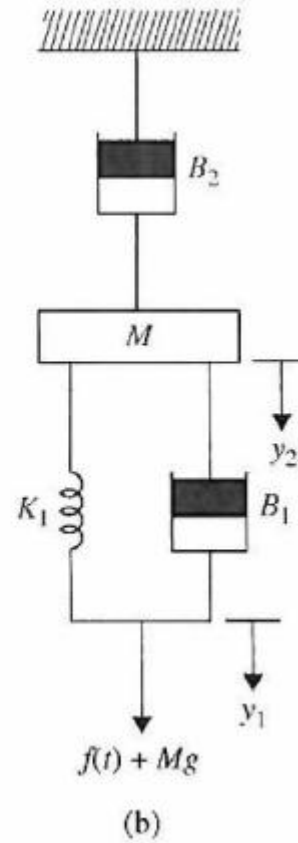
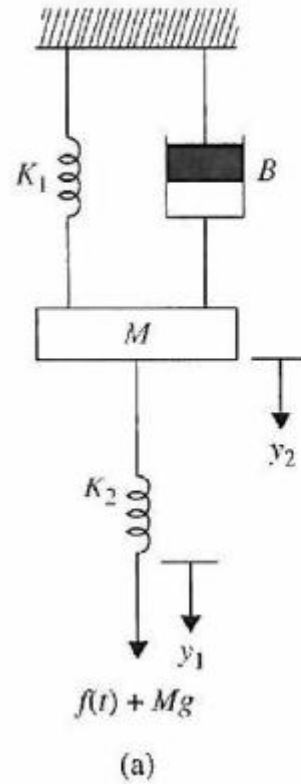
$$\dot{x}_1(t) = \frac{-1}{x_2^2(t)} \quad (4-266)$$

$$\dot{x}_2(t) = u(t)x_1(t) \quad (4-267)$$

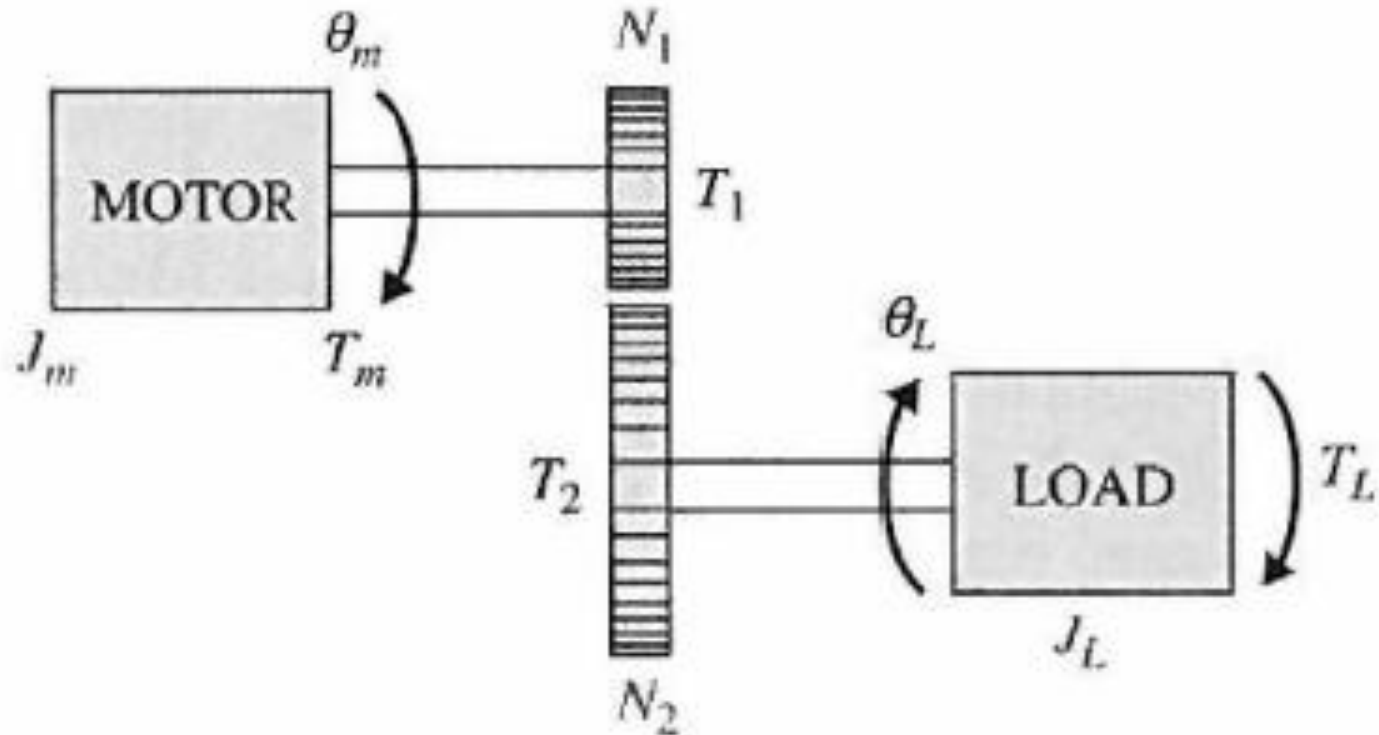
These equations are to be linearized about the nominal trajectory $[x_{01}(t), x_{02}(t)]$, which is the solution to the equations with initial conditions $x_1(0) = x_2(0) = 1$ and input $u(t) = 0$.

Linealizar para $t = 0$

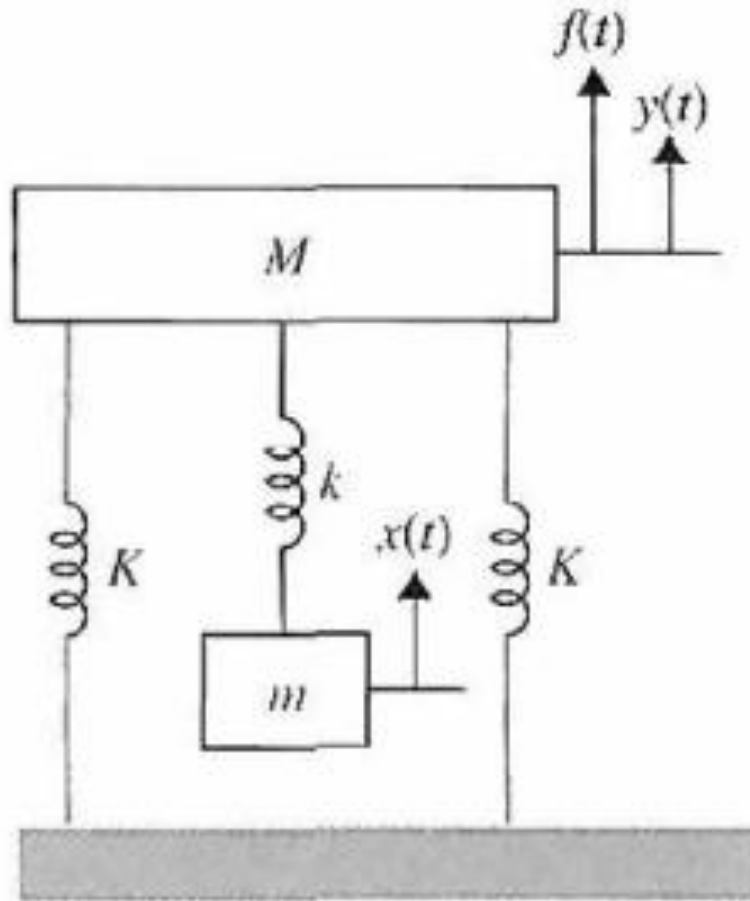
Ejercicios



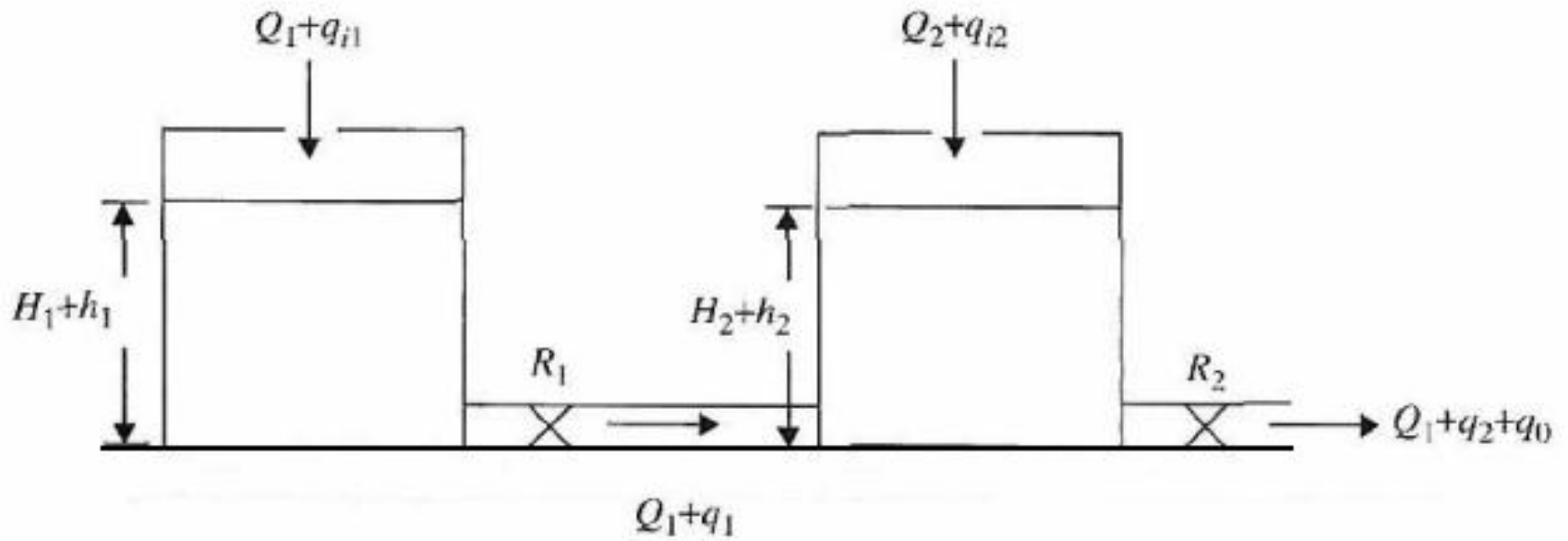
Ejercicios



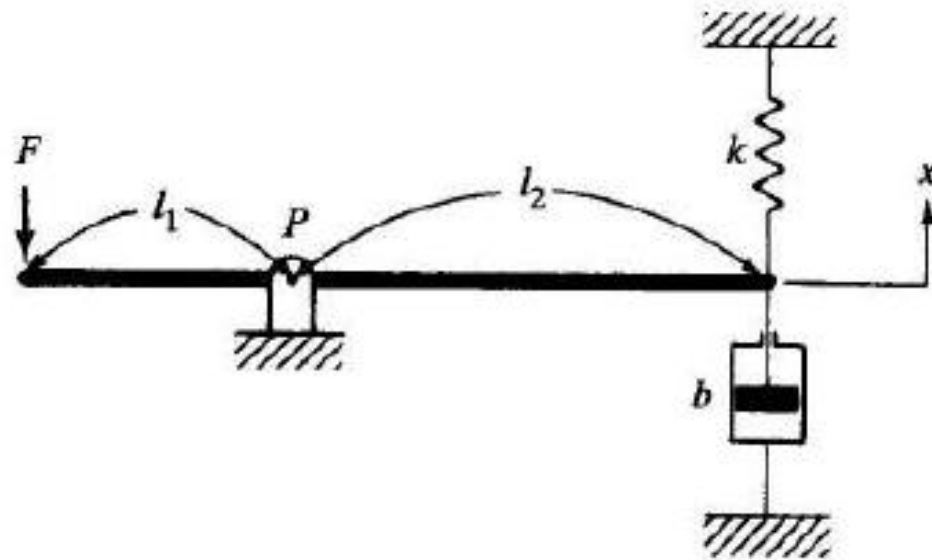
Ejercicios



Ejercicios



Ejercicios



In the mechanical system of Figure 3–28, one end of the lever is connected to a spring and a damper, and a force F is applied to the other end of the lever. Derive a mathematical model of the system. Assume that the displacement x is small and the lever is rigid and massless.

Ejercicios

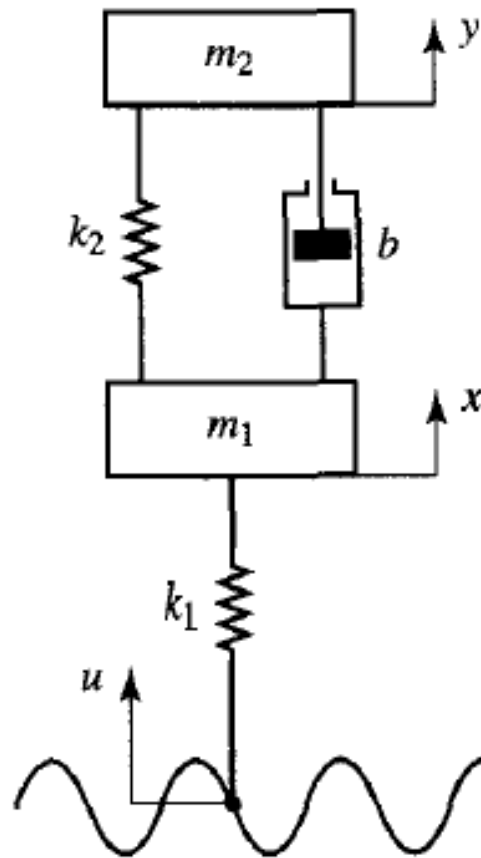
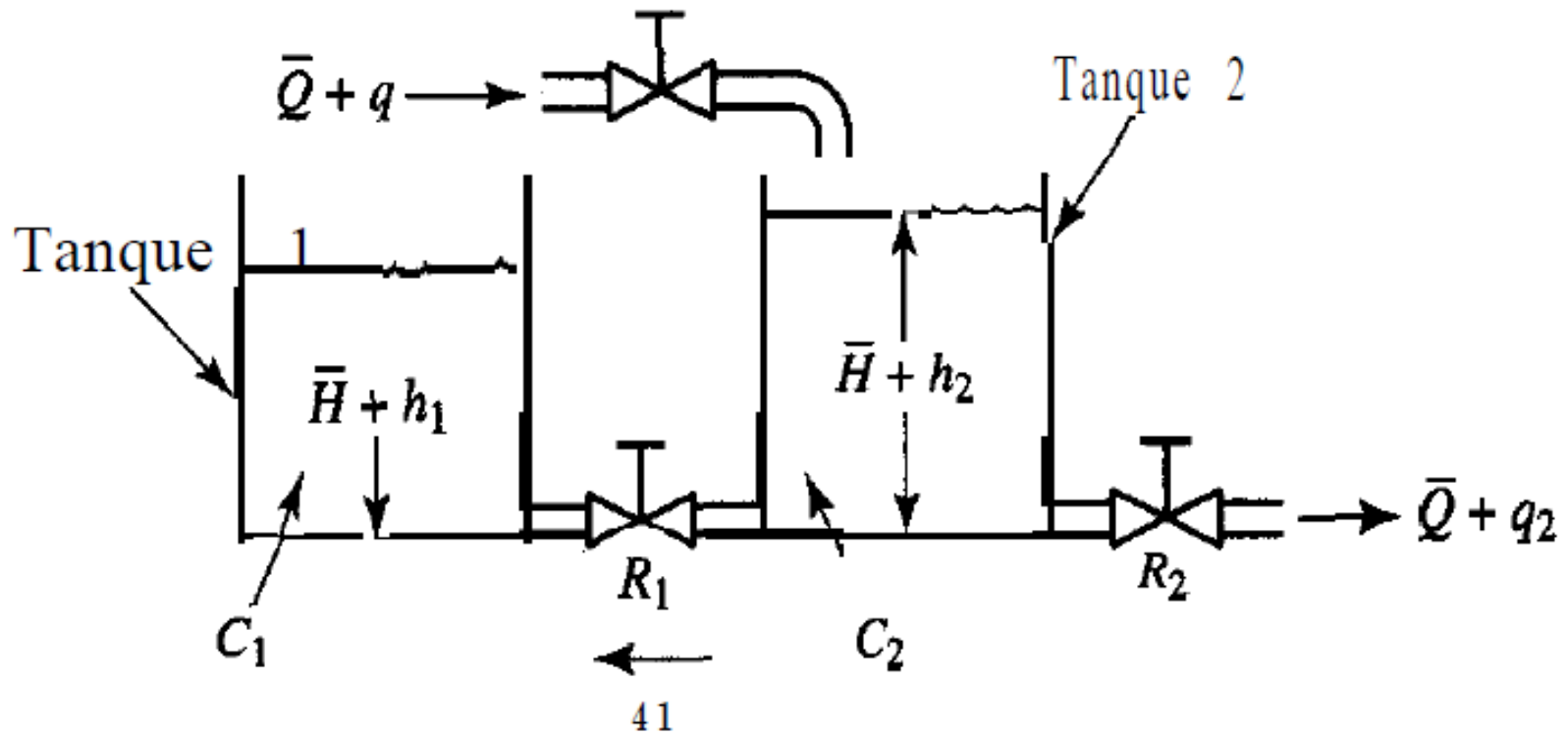


Figura 3-36
Sistema de suspensión.

Ejercicios



Ejercicios

Dado el sistema cuya ecuación diferencial es:

$$\dot{y} = 3x^2 + 4x \cdot \dot{x} + \operatorname{sen}(x) + 2$$

Linealizar en torno al punto de equilibrio dado por $x_0 = \pi/2$.